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SELF-SIMILAR SOLUTIONS FOR AN AXISYMMETRIC TURBULENT WAKE

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The problems of numerical simulation of axisymmetric turbulent wakes have been thoroughly described in [1]. However, no consideration was given to the widely used one-parameter turbulence models with respect to the coefficient of turbulent viscosity (see, for example, [2–7]). The present work partially fills this gap.

1. Statement of the Problem. Let us write the equation for a small defect of the velocity $u_d = U_\infty - u$ in a far axisymmetric wake [8] (an incompressible liquid with constant density):

$$U_\infty \frac{\partial u_d}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\,\varepsilon\,\frac{\partial u_d}{\partial r}\right). \tag{1.1}$$

Here the x axis coincides with the direction of the main-stream flow; U_∞ is the velocity of the main-stream flow; $r = (y^2 + z^2)^{1/2}$; ε is the coefficient of turbulent viscosity determined using a turbulence model. Three one-parameter turbulence models are considered for the case of a far axisymmetric wake:

the model of [4]

$$U_\infty \frac{\partial \varepsilon}{\partial x} = 0.6\,G\,\varepsilon\left|\frac{\partial u}{\partial r}\right| - b\left(\frac{\partial \varepsilon}{\partial r}\right)^2 + a\,G\,\frac{\varepsilon}{r}\frac{\partial}{\partial r}\left(r\,\frac{\partial \varepsilon}{\partial r}\right),$$

$$G = 1 + 100\,\frac{0.01\,F^4 + 10^{-10}}{0.01\,F^4 + 10^{-10} + (\partial u/\partial r)^4}, \quad F = \frac{1}{0.09}\left[\frac{1}{\varepsilon}\left(\frac{\partial \varepsilon}{\partial r}\right)^2 - 0.41^2\left|\frac{\partial u}{\partial r}\right|\right],$$

$$a = \frac{0.24}{0.25\cdot 0.41^2\cdot 0.4} = 14.277, \qquad b = \frac{0.24}{0.41^2\cdot 0.4} = 3.569;$$

the model of [5]

$$U_\infty \frac{\partial \varepsilon}{\partial x} = 0.1355\,\varepsilon\left|\frac{\partial u}{\partial r}\right| + 0.933\left(\frac{\partial \varepsilon}{\partial r}\right)^2 + \frac{1.5}{r}\frac{\partial}{\partial r}\left(r\,\varepsilon\,\frac{\partial \varepsilon}{\partial r}\right);$$

the model of [6]

$$U_\infty \frac{\partial \varepsilon}{\partial x} = 0.1\,\varepsilon\left|\frac{\partial u}{\partial r}\right| + 0.8\left(\frac{\partial \varepsilon}{\partial r}\right)^2 + \frac{0.8}{r}\frac{\partial}{\partial r}\left(r\,\varepsilon\,\frac{\partial \varepsilon}{\partial r}\right) - 0.05\,\varepsilon^{4/3}\left|\frac{1}{r}\frac{\partial}{\partial r}\left(r\,\frac{\partial u}{\partial r}\right)\right|^{2/3}$$
$$+0.4\left(\varepsilon\left|\frac{\partial u}{\partial r}\right|\right)^{1/2}\left|\frac{\partial \varepsilon}{\partial r}\right| + 4\,\varepsilon\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \varepsilon}{\partial r}\right) + \left|\frac{\partial^2 \varepsilon}{\partial r^2}\right|\right].$$

It should be noted that the constant 10^{-10} in the expression for G (model [4]) is introduced for convenience of calculations in order to avoid division by zero (see [4]). The following variables are introduced (see [9]):

$$f = \frac{u_d}{u_c}, \quad \eta = \frac{r}{l_c},$$

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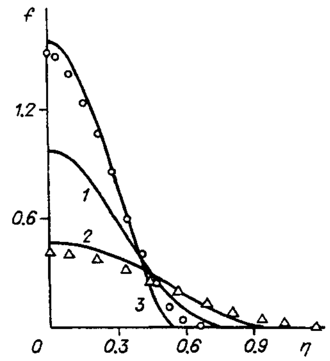

Fig. 1

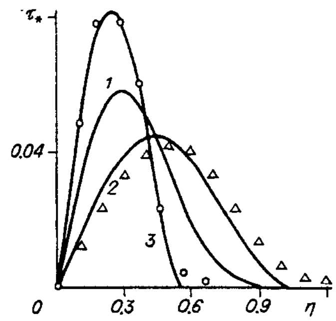

Fig. 2

where u_c and l_c are the characteristic scales of velocity and length, determined from the relations

$$u_c = U_\infty (x - x_0)^{-2/3}(C_x S)^{1/3}, \quad l_c = (C_x S)^{1/3}(x - x_0)^{1/3}.$$

Here $x_0 = \text{const}$; C_x is the drag coefficient of the streamlined body, which is determined with the aid of the characteristic body area S for which the frontal area is taken. Let us pass to the dimensionless coefficient of turbulent viscosity φ using the relation

$$\varphi = \varepsilon u_c^{-1} l_c^{-1}.$$

Assuming that $f = f(\eta)$ and $\varphi = \varphi(\eta)$, after transformations of Eq. (1.1), we obtain

$$\eta f' + 2f + 3\varphi f'' + 3 f'(\varphi' + \varphi\eta^{-1}) = 0.$$

The above turbulence models in the new notation take the following form:

the model of [4]

$$\eta \varphi' + \varphi + 1.8 G \varphi |f'| + 3 a G \varphi \varphi'' + 3\varphi'(a G \varphi \eta^{-1} - 3 b \varphi') = 0,$$

$$G = 1 + 100 \frac{0.01 F^4 + 10^{-10}}{0.01 F^4 + 10^{-10} + (f')^4}, \qquad F = \frac{1}{0.09}\left[\frac{\varphi' \varphi'}{\varphi} - 0.41^2 |f'|\right];$$

the model of [5]

$$\eta \varphi' + \varphi + 0.4065 \varphi |f'| + 4.5 \varphi \varphi'' + \varphi'(4.5 \varphi \eta^{-1} + 7.299 \varphi') = 0;$$

the model of [6]

$$\eta\varphi' + \varphi + 0.3\varphi|f'| + 14.4\varphi\varphi'' + 12\varphi|\varphi''| + \varphi'(14.4\varphi\eta^{-1} + 4.8\varphi') - 0.15\varphi^{4/3}|f'' + f'\eta^{-1}|^{2/3} + 1.2(\varphi|f'|)^{1/2}|\varphi'| = 0.$$

The boundary conditions are as follows:

$$f'(0) = f(\infty) = \varphi'(0) = \varphi(\infty) = 0.$$

We note that the flow in the wake satisfies the condition of equality of the excess momentum flow to the resistance force:

$$2\pi \int_0^\infty u (U_\infty - u) r \, dr = C_x 0.5 U_\infty^2 S.$$

Passing to self-similar variables in this relation, we obtain the normalization for f:

$$\int_0^\infty f \eta \, d\eta = (4\pi)^{-1}. \tag{1.2}$$

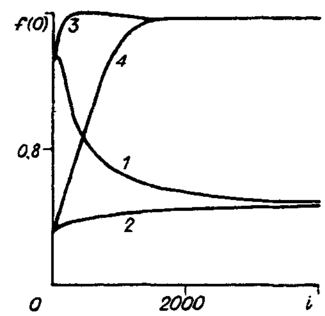

Fig. 3

2. Calculation Results. To solve the problem stated, we used the pseudo-nonstationary method and an implicit monotone finite-difference scheme [10] of the first order of accuracy. The calculations were carried out on a grid with number of nodes $N = 800$. A twofold diminution of the number of nodes resulted in a change in the values $f(0)$ and $\varphi(0)$ by a factor of -3.1 and $+0.8\%$, respectively, with the use of the model of [4], by a factor of $+0.3$ and -0.7% with the use of the model of [5], and -0.8 and $+0.06\%$ with the model of [6].

The distributions of the relative velocity $f(\eta)$ and relative friction stress $\tau_*(\eta) = \varphi \mid f' \mid$ in the wake were calculated. The dependences are presented in Figs. 1 and 2, where lines 1–3 correspond to the turbulence models of [4–6] and circles and triangles denote the experimental data of [9] for a prolate body of revolution and a sphere, respectively. It is evident that the calculations with the use of the turbulence model of [5] agree well with the experimental data obtained for a bluff body — a sphere.

Use of the model of [6] provides good agreement with the experimental data for a streamlined body — a prolate body of revolution. Finally, the calculations with the model [4] enable one to obtain results that are approximately equidistant from experimental data for both bluff and streamlined bodies.

In addition to the self-similar solutions shown in Figs. 1 and 2, there is a trivial solution $f = 0$, $\varphi = 0$. No other self-similar solutions were obtained. As an example, we consider two variants of specification of initial data necessary to apply the iteration technique of solving the problem stated (in the first variant initial data are close to the experimental results for a streamlined body: in the second, to those for a bluff body):

variant 1

$$f = \begin{cases} \frac{4}{\pi}(1 - 4\eta^2), & 0 \leqslant \eta \leqslant 0.5, \\ 0, & \eta > 0.5, \end{cases} \qquad \varphi = \begin{cases} \frac{(1-4\eta^2)}{30}, & 0 \leqslant \eta \leqslant 0.5, \\ 0, & \eta > 0.5; \end{cases}$$

variant 2

$$f = \begin{cases} \frac{(1-\eta^2)}{\pi}, & 0 \leqslant \eta \leqslant 1, \\ 0, & \eta > 1, \end{cases} \qquad \varphi = \begin{cases} 0.13(1 - \eta^2), & 0 \leqslant \eta \leqslant 1, \\ 0, & \eta > 1. \end{cases}$$

It should be noted that in both cases the normalization condition (1.2) holds. The change of $f(0)$ with iterations i is shown in Fig. 3, where lines 1 and 2 correspond to the turbulence model of [5], and lines 3 and 4, to the model of [6]; lines 1 and 3 correspond to the first variant, whereas 2 and 4, to the second variant. It is evident that the stationary values $f(0)$ are independent of the variant of initial data specification. A similar result is obtained for the model of [4].

The above calculation data suggest that the turbulence models considered here have a unique nontrivial solution and for this reason are unable to describe the experimentally observed effect of body shape on the characteristics of a far wake. The same situation is characteristic of the other well-known turbulence models. The above imperfection of the turbulence models should be taken into account in selecting a model for calculating flow behind a particular body. For a streamlined body, one can use the model of [6], and for a bluff body, the model of [5]. If the character of flow past a body cannot be foreseen, the model of [4] can provide approximate estimates.

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